

Unambiguous discrimination of nonorthogonal quantum states in cavity QED

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We propose an oversimplified scheme to unambiguously discriminate nonorthogonal quantum field states inside high-Q cavities. Our scheme, which is based on positive operator-valued measures (POVM) technique, uses a single three-level atom interacting resonantly with a single mode of a cavity-field and selective atomic state detectors. While the single three-level atom takes the role of the ancilla, the single cavity mode field represents the system we want to obtain information. The efficiency of our proposal is analyzed considering the nowadays achievements in the context of cavity QED.

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I. INTRODUCTION

Positive operator-valued measures (POVM) generalizes all possible kind of measurements [1, 2] and cannot be reduced to standard projections of the initial state onto orthogonal states spanning the initial Hilbert space alone, pertaining to the system we want to obtain information [3, 4]. In fact, although in general POVM can always be realized as standard projective measurements on an enlarged system [3], they are such that the number of outputs may be larger than the dimensionality of the space of states of the system in which we are interest in. POVM is now standard in several areas of quantum mechanics, including quantum optics and quantum information, among others [4–7]. In this paper we show how to accomplish POVM to unambiguously discriminate nonorthogonal field states inside high-Q cavities. The goal of unambiguous quantum state discrimination (UQSD) is to discern in which state the system was prepared [8–11], founding many applications in several protocols [9, 10], mainly for quantum cryptography [12–15]. Our scheme, employing one three-level atom interacting with a single mode of a cavity field, is very simple from the experimental point of view and can be implemented using nowadays techniques in cavity QED.

We begin by reviewing the general quantum measurement theory. Next, we present our model and results, comparing with the simple case of projective measurements. Then we present our conclusions.

II. GENERAL MEASUREMENTS

Consider a quantum system we are interested to measure, and a second quantum system we call the *ancilla*, which is used to get information about the system of interest [2]. Let the Hilbert space dimension of the system of interest and the ancilla as K and L , respectively. The ancilla is prepared in some known initial state independently of the system of interest, and then the two systems are allowed to interact, getting correlated. Next,

a von Neumann measurement is performed on the ancilla, providing us with information about the system of interest, which we are going to call the system from now on. Let us call the initial state of the ancilla as $|a_0\rangle$ in the basis $\{|a_k\rangle\}$, $k = 0, 1, \dots, K - 1$, and denote the initial density operator of ancilla-system as

$$\rho_{AS} = |a_0\rangle\langle a_0| \otimes \rho_S, \quad (1)$$

where ρ_S is the initial density operator of the system. Let U denote the ancilla-system evolution operator. Since U acts in the tensor-product space, it can be written as

$$U = \sum_{kk'} |a_k\rangle\langle a_{k'}| \otimes M_{kk'} \quad (2)$$

where

$$M_{kk'} = \sum_{ll'} u_{klk'l'} |s_l\rangle\langle s_{l'}|, \quad (3)$$

being $\{|s_l\rangle\}$, $l = 0, 1, \dots, L - 1$, a set of basis states for the system, and $u_{klk'l'} = \langle a_k, s_l | U | a_{k'}, s_{l'} \rangle$ are the matrix elements of U . Note that $M_{kk'}$ acts in the Hilbert space of the system, and since the space of system has dimension L , each sub-block matrix $M_{kk'}$ has dimension L . From now on we use M_k to refer to the first column of the sub-block M_{k0} of U . Denoting the $K \times K$ sub-blocks of the matrix $U^\dagger U$ by $B_{kk'}$, and since $U^\dagger U = I$, it is readily seen that

$$B_{00} = \sum_k M_k^\dagger M_k = I. \quad (4)$$

The important point to note here is that M_k can be chosen to be any set of operators, provided the restriction Eq. (4) above is obeyed.

Now, performing a von Neumann measurement on the ancilla states, represented by $|a_m\rangle\langle a_m|$, we can write the (unnormalized) collapsed state of both ancilla and system as

$$\tilde{\rho}_{AS,m} = (|a_m\rangle\langle a_m| \otimes I) \rho_{AS} (|a_m\rangle\langle a_m| \otimes I) \quad (5)$$

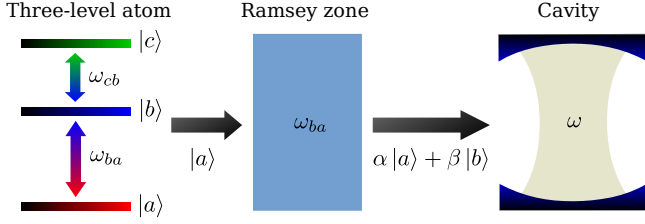


Figure 1. Scheme of the experimental setup to implement POVM in cavity QED. A three-level atom interacts with a single mode of a high Q cavity. The cavity mode field is prepared in one of two nonorthogonal states $|\psi_1\rangle$ and $|\psi_2\rangle$. Our POVM discriminates between these two nonorthogonal states prepared inside the high Q cavity.

or, in terms of the sub-blocks of U :

$$\tilde{\rho}_{AS,m} = |a_m\rangle \langle a_m| \otimes M_m \rho_S M_m^\dagger. \quad (6)$$

From Eq. (6) we can write the normalized state of the system as

$$\tilde{\rho}_{S,m} = \frac{M_m \rho_S M_m^\dagger}{p_m}, \quad (7)$$

where

$$p_m = \text{Tr}(M_m^\dagger M_m \rho_S) \quad (8)$$

is the probability of finding the ancilla in state $|a_m\rangle$ after the unitary evolution U . It is now promptly recognized that every set of operators $\{M_k\}$ satisfying $\sum_k M_k^\dagger M_k \equiv \sum_k E_k = I$ describes a possible measurement on a quantum system, with the measuring having K outcomes.

This gives us a complete description of a quantum system under a general measurement. Next, we use the above results to discriminate one of two nonorthogonal field states prepared into a high Q cavity.

III. MODEL

In our proposal, see Fig. 1, a three-level atom in ladder configuration, described by the set of states $\{|a\rangle, |b\rangle, |c\rangle\}$, is initially prepared in $|a\rangle$ and crosses a Ramsey zone (carrier interaction). Next, the atom enters a cavity interacting on-resonance with a single mode of a cavity field which in turn is prepared either in state $|\psi_1\rangle$ or in state $|\psi_2\rangle$, for which $\langle\psi_1|\psi_2\rangle \neq 0$ (nonorthogonal states). While inside the cavity, the atom suffers a Stark shift in order to lead $\omega_{ba} = \omega_{cb} = \omega$ [16, 17]. After the atom crosses the cavity, it is detected in one of its three possible states, thus revealing in which state the cavity mode was prepared. The Hamiltonian model is given by [5]

$$H = \hbar g_1 (\sigma_{ba} a + \sigma_{ab} a^\dagger) + \hbar g_2 (\sigma_{cb} a + \sigma_{bc} a^\dagger), \quad (9)$$

where $\sigma_{\eta\nu} = |\eta\rangle \langle \nu|$ and a^\dagger (a) is the creation (annihilation) photon number operator, and g_i is the atom-field coupling, which we take as real for convenience. In this protocol we are interested in discriminating nonorthogonal states which are combinations of the Fock states $|0\rangle$ and $|1\rangle$. Thus, since the maximum number of photons in this case is $n = 2$, which happens when the atom decays and increases the photon number into the cavity, we can consider $a^\dagger |2\rangle = 0$. After a little algebra, it is straightforward to obtain

$$U |a, 0\rangle = \alpha |a, 0\rangle - i\beta \sin(g_1 t) |a, 1\rangle + \beta \cos(g_1 t) |b, 0\rangle, \quad (10)$$

$$U |a, 1\rangle = \alpha \cos(g_1 t) |a, 1\rangle - i\beta \left(\frac{\sqrt{2}g_1}{\sqrt{2g_1^2 + g_2^2}} \right) \sin\left(\sqrt{2g_1^2 + g_2^2} t\right) |a, 2\rangle - i\alpha \sin(g_1 t) |b, 0\rangle + \\ + \beta \cos\left(\sqrt{2g_1^2 + g_2^2} t\right) |b, 1\rangle - i\beta \left(\frac{g_2}{\sqrt{2g_1^2 + g_2^2}} \right) \sin\left(\sqrt{2g_1^2 + g_2^2} t\right) |c, 0\rangle, \quad (11)$$

$$U |a, 2\rangle = \alpha \left\{ 1 + \left(\frac{2g_1^2}{2g_1^2 + g_2^2} \right) \left[\cos\left(\sqrt{2g_1^2 + g_2^2} t\right) - 1 \right] \right\} |a, 2\rangle - i\alpha \left(\frac{\sqrt{2}g_1}{\sqrt{2g_1^2 + g_2^2}} \right) \sin\left(\sqrt{2g_1^2 + g_2^2} t\right) |b, 1\rangle + \\ + \beta \cos\left(\sqrt{2g_1^2 + g_2^2} t\right) |b, 2\rangle + \alpha \left(\frac{\sqrt{2}g_1 g_2}{2g_1^2 + g_2^2} \right) \left[\cos\left(\sqrt{2g_1^2 + g_2^2} t\right) - 1 \right] |c, 0\rangle - i\beta \sin\left(\sqrt{2g_1^2 + g_2^2} t\right) |c, 1\rangle, \quad (12)$$

where $U = U_C U_{RZ}$, with $U_C = e^{-iHt/\hbar}$, H given by

Eq.(9), and U_{ZR} is the evolution operator as given by

the carrier or Ramsey zone: $U_{RZ} |a\rangle = \alpha |a\rangle + \beta |b\rangle$.

Following the standard procedure [4], we must build three POVM elements: the one that discriminates $|\psi_1\rangle$; the other one that discriminates $|\psi_2\rangle$, and a third one leading to inconclusive results with probability p_{in} . It is to be noted that the only constraint obeyed by the POVM elements is $\sum_k E_k = I$. Thus, as soon as the atom state is known, we will know with certainty that the cavity mode field was either in state $|\psi_1\rangle$ or in state $|\psi_2\rangle$, or that we do not know the initial state as a result of

the inconclusive measurement. As explained above, to build the three POVM elements $E_\nu = M_\nu^\dagger M_\nu$, $\nu = a, b, c$, we have to calculate $(n, n' = 1, 2, 3)$:

$$M_\nu = \sum_{nn'} u_{\nu nan'} |n\rangle \langle n'|, \quad (13)$$

and

$$u_{\nu nan'} = \langle \nu, n | U | a, n' \rangle. \quad (14)$$

Using Eq. (10)-(12), we calculate the following operators in Eq. (13):

$$M_a = \alpha |0\rangle \langle 0| - i\beta \sin(g_1 t) |1\rangle \langle 0| + \alpha \cos(g_1 t) |1\rangle \langle 1| - i\beta \left(\frac{\sqrt{2}g_1}{\sqrt{2g_1^2 + g_2^2}} \right) \sin\left(\sqrt{2g_1^2 + g_2^2}t\right) |2\rangle \langle 1| + \alpha \left\{ 1 + \left(\frac{2g_1^2}{2g_1^2 + g_2^2} \right) \left[\cos\left(\sqrt{2g_1^2 + g_2^2}t\right) - 1 \right] \right\} |2\rangle \langle 2|, \quad (15)$$

$$M_b = \beta \cos(g_1 t) |0\rangle \langle 0| - i\alpha \sin(g_1 t) |0\rangle \langle 1| + \beta \cos\left(\sqrt{2g_1^2 + g_2^2}t\right) |1\rangle \langle 1| - i\alpha \left(\frac{\sqrt{2}g_1}{\sqrt{2g_1^2 + g_2^2}} \right) \sin\left(\sqrt{2g_1^2 + g_2^2}t\right) |1\rangle \langle 2| + \beta \cos(\sqrt{2}g_2 t) |2\rangle \langle 2|, \quad (16)$$

$$M_c = -i\beta \left(\frac{g_2}{\sqrt{2g_1^2 + g_2^2}} \right) \sin\left(\sqrt{2g_1^2 + g_2^2}t\right) |0\rangle \langle 1| + \alpha \left(\frac{\sqrt{2}g_1 g_2}{2g_1^2 + g_2^2} \right) \left[\cos\left(\sqrt{2g_1^2 + g_2^2}t\right) - 1 \right] |0\rangle \langle 2| - i\beta \sin(\sqrt{2}g_2 t) |1\rangle \langle 2|. \quad (17)$$

From Eq. (15)-(17) we can calculate the POVM elements

$$E_\nu = M_\nu^\dagger M_\nu \text{ for } \nu = a, b, c:$$

$$E_a = \left[|\alpha|^2 + |\beta|^2 \sin^2(g_1 t) \right] |0\rangle \langle 0| + i \sin(g_1 t) \cos(g_1 t) (\alpha\beta^* |0\rangle \langle 1| - \alpha^* \beta |1\rangle \langle 0|) + \left[|\alpha|^2 \cos^2(g_1 t) + |\beta|^2 \left(\frac{2g_1^2}{2g_1^2 + g_2^2} \right) \sin^2\left(\sqrt{2g_1^2 + g_2^2}t\right) \right] |1\rangle \langle 1| + i \left(\frac{\sqrt{2}g_1}{\sqrt{2g_1^2 + g_2^2}} \right) \left\{ 1 + \left(\frac{2g_1^2}{2g_1^2 + g_2^2} \right) \left[\cos\left(\sqrt{2g_1^2 + g_2^2}t\right) - 1 \right] \right\} (\alpha\beta^* |1\rangle \langle 2| - \alpha\beta^* |2\rangle \langle 1|) + |\alpha|^2 \left\{ 1 + \left(\frac{2g_1^2}{2g_1^2 + g_2^2} \right) \left[\cos\left(\sqrt{2g_1^2 + g_2^2}t\right) - 1 \right] \right\}^2 |2\rangle \langle 2|, \quad (18)$$

$$\begin{aligned}
E_b = & |\beta|^2 \cos^2(g_1 t) |0\rangle \langle 0| - i \sin(g_1 t) \cos(g_1 t) (\alpha \beta^* |0\rangle \langle 1| - \alpha \beta^* |1\rangle \langle 0|) + \\
& + \left[|\alpha|^2 \sin^2(g_1 t) + |\beta|^2 \cos^2\left(\sqrt{2g_1^2 + g_2^2} t\right) \right] |1\rangle \langle 1| - \\
& - i \left(\frac{\sqrt{2}g_1}{\sqrt{2g_1^2 + g_2^2}} \right) \sin\left(\sqrt{2g_1^2 + g_2^2} t\right) \cos\left(\sqrt{2g_1^2 + g_2^2} t\right) (\alpha \beta^* |1\rangle \langle 2| - \alpha \beta^* |2\rangle \langle 1|) + \\
& + \left[|\alpha|^2 \left(\frac{2g_1^2}{2g_1^2 + g_2^2} \right) \sin^2\left(\sqrt{2g_1^2 + g_2^2} t\right) + |\beta|^2 \cos^2\left(\sqrt{2}g_2 t\right) \right] |2\rangle \langle 2|, \quad (19)
\end{aligned}$$

$$\begin{aligned}
E_c = & |\beta|^2 \left(\frac{g_2^2}{2g_1^2 + g_2^2} \right) \sin^2\left(\sqrt{2g_1^2 + g_2^2} t\right) |1\rangle \langle 1| + \\
& + i \left[\frac{\sqrt{2}g_1 g_2^2}{(2g_1^2 + g_2^2)^{\frac{3}{2}}} \right] \sin\left(\sqrt{2g_1^2 + g_2^2} t\right) \left[\cos\left(\sqrt{2g_1^2 + g_2^2} t\right) - 1 \right] (\alpha \beta^* |1\rangle \langle 2| - \alpha \beta^* |2\rangle \langle 1|) + \\
& + \left\{ |\alpha|^2 \left[\frac{2g_1^2 g_2^2}{(2g_1^2 + g_2^2)^2} \right] \left[\cos\left(\sqrt{2g_1^2 + g_2^2} t\right) - 1 \right]^2 + |\beta|^2 \sin^2\left(\sqrt{2}g_2 t\right) \right\} |2\rangle \langle 2|. \quad (20)
\end{aligned}$$

As can be checked, $\sum_\nu E_\nu = I$.

To be specific, let us assume that we want to discriminate the following nonorthogonal field states into the cavity: $|\psi_1\rangle = |0\rangle$ and $|\psi_2\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ [4]. The cavity state is thus represented by $\rho = q_1 |\psi_1\rangle \langle \psi_1| + q_2 |\psi_2\rangle \langle \psi_2|$, where q_1 (q_2) is the classical probability related to the frequency of preparing the state $|\psi_1\rangle$ ($|\psi_2\rangle$) and $q_1 + q_2 = 1$. Clearly, E_c discriminates state $|\psi_2\rangle$, since $\langle \psi_1 | E_c | \psi_1 \rangle = 0$. To discriminate $|\psi_1\rangle$, we impose $\langle \psi_2 | E_b | \psi_2 \rangle = 0$. This imposition leads us with the conditions: (i) $\alpha = \cos(g_1 t)$, (ii) $\beta = i \sin(g_1 t)$, and (iii) $\cos\left(\sqrt{2g_1^2 + g_2^2} t\right) = 0$. Letting $g_2 = \kappa g_1$, the third condition can be rewritten as $g_1 t \equiv \theta_m = \frac{(m+\frac{1}{2})\pi}{\sqrt{2+\kappa^2}}$, $m = 0, 1, 2, \dots$. On the other hand, E_a is inconclusive, since $\langle \psi_1 | E_a | \psi_1 \rangle \neq 0$ and $\langle \psi_2 | E_a | \psi_2 \rangle \neq 0$, meaning that we must discard this measurement. Using these three conditions, we can write the effective POVM elements in the following way:

$$\begin{aligned}
E_a = & (\cos^2 \theta_m + \sin^4 \theta_m) |0\rangle \langle 0| \\
& + \frac{1}{4} \sin^2(2\theta_m) (|0\rangle \langle 1| + |1\rangle \langle 0|) + \\
& + \left[\cos^4 \theta_m + \left(\frac{2}{2 + \kappa^2} \right) \sin^2 \theta_m \right] |1\rangle \langle 1|, \quad (21)
\end{aligned}$$

$$E_b = \frac{1}{2} \sin^2(2\theta_m) |\psi_2^\perp\rangle \langle \psi_2^\perp|, \quad (22)$$

$$E_c = \left(\frac{\kappa^2}{2 + \kappa^2} \right) \sin^2 \theta_m |1\rangle \langle 1|, \quad (23)$$

where we have neglected terms containing the state $|2\rangle$ and put $|\psi_2^\perp\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$.

The probabilities related to the success probability rates of POVM elements E_b and E_c are, respectively,

$$p_b = \text{Tr}(E_b \rho) = \frac{q_1}{4} \sin^2(2\theta_m) \quad (24)$$

and

$$p_c = \text{Tr}(E_c \rho) = \frac{q_2}{2} \left(\frac{\kappa^2}{2 + \kappa^2} \right) \sin^2(\theta_m), \quad (25)$$

while the probability for the inconclusive results $p_{in} = p_a = \text{Tr}(E_a \rho) = q_1 \langle \psi_1 | E_a | \psi_1 \rangle + q_2 \langle \psi_2 | E_a | \psi_2 \rangle$ is

$$\begin{aligned}
p_{in} = & q_1 (\cos^2 \theta_m + \sin^4 \theta_m) + \\
& + \frac{q_2}{2} \left[1 + \cos^2 \theta_m + \left(\frac{2}{2 + \kappa^2} \right) \sin^2 \theta_m \right]. \quad (26)
\end{aligned}$$

The success probability is given by $p_s = p_b + p_c$:

$$p_s = \frac{q_1}{4} \sin^2(2\theta_m) + \frac{q_2}{2} \left(\frac{\kappa^2}{2 + \kappa^2} \right) \sin^2(\theta_m), \quad (27)$$

where $p_s + p_{in} = 1$, as should.

IV. DISCUSSION

Since the result of POVM element E_a is the inconclusive one, all we have to do in order to optimize our proposal is either minimize p_{in} or maximize p_s . We numerically maximize Eq. (27) with (i) $q_1 = 0.3$, (ii) $q_1 = 0.7$

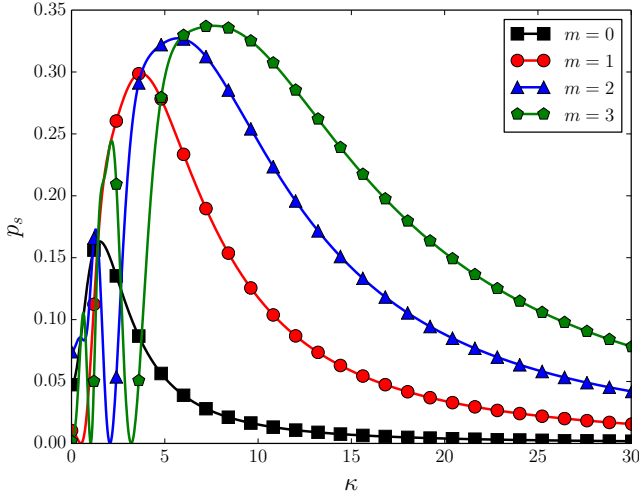


Figure 2. Success probability p_s versus the coupling ratio $\kappa = g_2/g_1$ for $q_1 = 0.3$. The best choice to UQED is the one that maximizes p_s to each integer m .

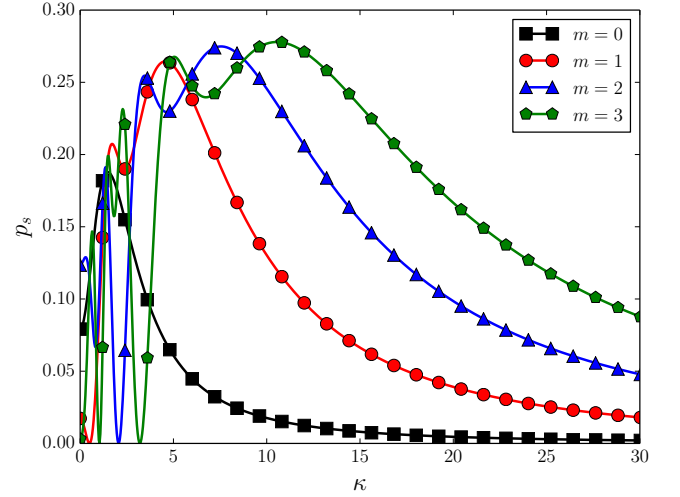


Figure 3. Success probability p_s versus the coupling ratio $\kappa = g_2/g_1$ for $q_1 = 0.7$. The best choice to UQED is the one that maximizes p_s to each integer m .

and, for comparison to other work, (iii) $q_1 = q_2 = 0.5$ [4]. As an example, to $q_1 = q_2 = 0.5$ we find, see Fig. 4, (a) for $m = 0$, $p_s = 0.1878$ (black line with squares), implying $\kappa = 1, 47$, (b) for $m = 1$, $p_s = 0.2644$ (red line with circles), implying $\kappa = 4.50$ (c) for $m = 2$, $p_s = 0.2748$ (blue line with triangles), implying $\kappa = 7, 50$ (d) for $m = 3$, $p_s = 0.2779$ (green line pentagons), implying $\kappa = 10.55$. Values of $m \geq 3$ can be used at the expense of greater ratio $\kappa = g_2/g_1$, see Tab. I-III. The best choice to UQSD is the one that minimizes (maximizes) p_{in} (p_s) for each integer m , and it is worthwhile to mention that the success probability rate around 0.26, obtained for the first values of the integer m , is very close to the best rate of success for this kind of quantum state discrimination predicted theoretically, which is 0.292 when $q_1 = q_2$ [4, 7]. As can be seen from Fig. 2-3, there are several maxima in p_s , depending on the value of κ . Here we have chosen those whose success probability is the greatest one. Note from Tab. III, corresponding to $q_1 = q_2 = 0.5$, that the best value for the success probability rate is below 0.292, which is the maximum value according to Ref. [4, 7]. This is also confirmed by our numerical calculations using much greater values for m and κ .

It is to be noted that the simple strategy of choosing whether to project the cavity field state on the computational basis $|0\rangle$ or $|1\rangle$ would allow us to discriminate only one state. Indeed, if the result of projection is $|1\rangle$, the cavity mode state could not have been prepared in $|\psi_1\rangle$ and was prepared therefore in state $|\psi_2\rangle$; however, if the measurement result is $|0\rangle$, we can not be sure if the cavity mode state had been prepared in $|\psi_1\rangle$ or $|\psi_2\rangle$, this result being inconclusive. As a result

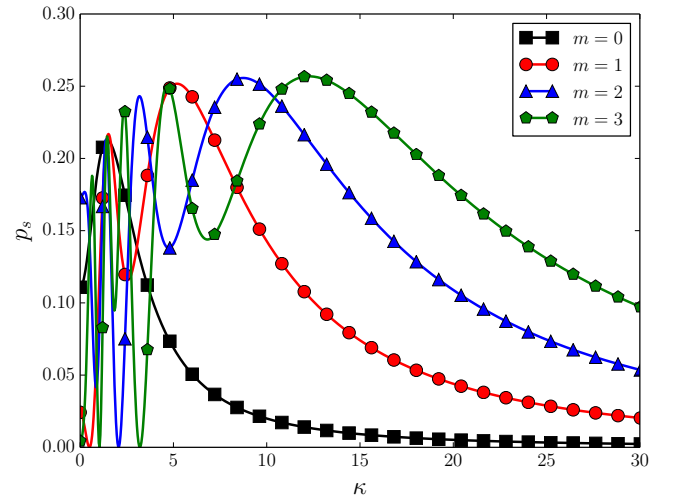


Figure 4. Success probability p_s versus the coupling ratio $\kappa = g_2/g_1$ for $q_1 = 0.5$. The best choice to UQED is the one that maximizes p_s to each integer m .

of this strategy, we would find a success probability of $p_s = \text{Tr}(|1\rangle\langle 1|\rho) = 0.25$, thus lesser than the POVM strategy we developed. In Tab. I-III we present the values of κ maximizing p_s and the corresponding values for p_{in} , p_b , p_c and p_s , Eq. (24)-(27), for several integers m and $q_1 = 0.3, 0.7, 0.5$.

As a final remark, one could ask why to use POVM strategy instead of simply projecting on the computational basis of the cavity states. Three remarks are in order: (i) First, in addition to our protocol presenting a higher probability of success, there is no known tech-

m	κ	p_{in}	p_b	p_c	p_s
0	1.52	0.8369	0.0747	0.0884	0.1631
1	3.72	0.7011	0.0366	0.2623	0.2989
2	5.71	0.6728	0.0155	0.3117	0.3272
3	7.63	0.6627	0.0069	0.3304	0.3373
4	9.54	0.6581	0.0032	0.3387	0.3419
5	11.44	0.6556	0.0015	0.3429	0.3444
10	21.23	0.6505	0.0012	0.3483	0.3495
20	41.11	0.6505	0.0001	0.3494	0.3495
50	101.05	0.6501	0.0000	0.3499	0.3499

Table I. Values for the probabilities according to our protocol to accomplish UQSD in cavity QED. To each integer m , there is a minimum for the inconclusive events p_{in} and a corresponding maximum for the probability of success p_s . The table shows p_{in} , p_b , p_c , and p_s separately separately for $p_1 = 0.3$.

m	κ	p_{in}	p_b	p_c	p_s
0	1.45	0.7874	0.1749	0.0377	0.2126
1	5.19	0.7483	0.1693	0.0824	0.2517
2	8.54	0.7443	0.1679	0.0878	0.2557
3	12.27	0.7432	0.1674	0.0894	0.2568
4	15.80	0.7427	0.1673	0.0900	0.2573
5	19.32	0.7426	0.1671	0.0903	0.2574
10	36.91	0.7422	0.1670	0.0908	0.2578
20	72.08	0.7420	0.1670	0.0910	0.2580
50	177.58	0.7420	0.1670	0.0910	0.2580

Table II. Values for the probabilities according to our protocol to accomplish UQSD in cavity QED. To each integer m , there is a minimum for the inconclusive events p_{in} and a corresponding maximum for the probability of success p_s . The table shows p_{in} , p_b , p_c , and p_b separately separately for $p_1 = 0.7$.

m	κ	p_{in}	p_b	p_c	p_s
0	1.47	0.8123	0.1248	0.0629	0.1877
1	4.50	0.7356	0.1039	0.1605	0.2644
2	7.55	0.7252	0.0989	0.1759	0.2748
3	10.55	0.7221	0.0957	0.1822	0.2779
4	13.70	0.7209	0.0956	0.1835	0.2791
5	16.50	0.7101	0.0950	0.1849	0.2799
10	31.52	0.7191	0.0941	0.1868	0.2809
20	61.50	0.7189	0.0938	0.1873	0.2811
50	151.50	0.7188	0.0930	0.1882	0.2812

Table III. Values for the probabilities according to our protocol to accomplish UQSD in cavity QED. To each integer m , there is a minimum for the inconclusive events p_{in} and a corresponding maximum for the probability of success p_s . The table shows p_{in} , p_b , p_c , and p_s separately separately for $p_1 = 0.5$.

nique to directly project the cavity state on the computational basis: usually, measurement of the cavity state requires additional atoms and/or cavities, thus being necessary to build another POVM elements to measure the cavity mode field [18, 19], (ii) second, the direct projective strategy does not discriminate both states but just $|\psi_1\rangle = |0\rangle$, while the POVM strategy allows us to discriminate both $|\psi_1\rangle$ and $|\psi_2\rangle$, (iii) the POVM strategy was build using known matter-radiation interaction parameters: it remains an open question if other types of interaction, such as those developed by effective Hamiltonian techniques [20–22], could attain optimal POVM results [23].

V. CONCLUSION

We have proposed an oversimplified scheme to build POVM elements allowing to discriminate nonorthogonal field states inside a high Q cavity. Besides to circumvent the impossibility to directly project the cavity states onto the computational Fock states without using ancilla, our protocol achieves a rate of success probability greater than the direct projective technique. Our proposal relies on nowadays techniques in the cavity QED domain, making use of just one single three-level atom undergoing a Ramsey zone (carrier interaction) plus one cavity and selective atomic state detectors. This simplicity makes our protocol very attractive from the experimental point of view. Finally, we hope our protocol can inspire other POVM strategies based on effective Hamiltonians technique making possible to attain optimal rates of success probability.

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